# Repeated Moral Hazard with Private Evaluation: Leniency Bias

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**Abstract**

We study an infinitely repeated moral hazard problem in which the principal privately observes and publicly reports the agent's output, as in Fuchs (2007). The optimal perfect public equilibrium is constructed. We show that (i) the principal overreports the agent's output, known as leniency bias; (ii) as the discount factor goes to one, the probability with which the principal overreports the agent's output goes to one as well; (iii) more costly for the agent to exert effort, less likely for the principal to overreport the agent's output. Moreover, the optimal contract is shown to be static and self-enforcing.

**Keywords**: Moral Hazard, Private Evaluation, Leniency Bias

**JEL Classifications**: C73, D86

# 1 Introduction

Many firms assess and reward their employees based on subjective performance measures. Macleod and Parent (1999) report that more than 20 percent of U.S. workers receive subjectively determined rewards each year. Subjective performance evaluations have resulted in interesting insights into how incentives work in theory and practice, as discussed by Prendergast (1999). Compared to the objective evaluations, subjective evaluations are essentially private and non-verifiable by outsiders. Hence, employers have incentives to underreport employees' performance in order to save on wages/bonuses.

However, this intuition appears to be inconsistent with empirical evidence, which indicates that employees always get their evaluations leniently. For instance, Murphy (1992) observes that 95 percent of workers have gotten their appraisals above the average mark. Moers (2005) and Breuer et al. (2013) find strong evidence that subjective evaluations are upwards biased and employees are given better ratings. The appraisals are skewed to the top end of the rating scale. This phenomenon is called leniency bias – a tendency to provide employees with inflated performance ratings.

To better understand the observations in empirical studies, some interesting questions are put forth. Why do employers report subjective performance leniently? Can leniency bias emerge as an equilibrium outcome of the optimal contract? What determines the magnitude of leniency bias? Our goal in this paper is to characterize the optimal contract in order to answer these questions.

We consider an infinitely repeated moral hazard problem in which the principal privately observes, then publicly reports the agent's output. Both parties are risk neutral such that the total payoff is used to measure efficiency. We explicitly solve for the optimal perfect public equilibrium without any constraint such as the principal has to report truthfully as in Levin (2003), or the agent has to use a pure strategy as in Fuchs (2007), and for any given discount factor.

We first generalize the method of Radner et al. (1986) by taking into consideration the minmax payoffs to derive an upper bound on the maximum total payoff attainable by perfect public equilibria (thereafter PPE). The upper bound is not completely independent of the discount factor. Specifically, there is a threshold below which the unique PPE is the repetition of Nash equilibrium in the stage game where the agent always shirks and the principal always reports the low output.

For any discount factor above the threshold, we construct the optimal PPE which attains the upper bound. Specifically, the principal always reports the agent's high output truthfully, but only reports the agent's low output truthfully with some probability. That is, with some probability, the principal reports the high output even when the agent's output is indeed low. Hence, there exists a tendency to inflate the agent's performance. As suggested by Prendergast (1999), this phenomenon is well documented in empirical studies as leniency bias.

Compared to the case with public evaluation in which the agent's incentives to exert effort monotonically increase with the bonus for the high output, the agent's incentives to exert effort are non-monotonic in the bonus. The reason is as follows. One one hand, if the bonus is too small, then the agent has no incentives to exert effort, which is costly. On the other hand, if the bonus is too large, then the principal has incentives to underreport the agent's output to avoid paying the bonus such that the agent again has no incentives to exert effort. Therefore, there is a tradeoff between the agent's incentives to exert effort and principal's incentives to report truthfully.

We show that the optimal contract is static and self-enforcing. The principal has to report the agent's low output truthfully at least with some probability in order to provide incentives for the agent to exert effort. At the same time, the agent has to punish the principal for reporting the low output by shirking subsequently in order to provide incentives for the principal to report truthfully. Hence, the principal would report the agent's low output with a probability just large enough such that the agent has incentives to exert effort, but not too large to trigger unnecessary retaliation from the agent. Interestingly, with subjective evaluation, the agent would exert effort for a bonus too small (such that he would certainly shirk in the case of public evaluation) in order to provide incentives for the principal to be truthfully.

Specifically, if it becomes more costly for the agent to exert effort, then the principal has to express less leniency in order to motivate the agent to exert (more costly) effort. If the agent becomes more patient, then the principal would express more leniency to avoid retaliation without demotivating the (more patient) agent. Furthermore, as the discount factor goes to one, leniency bias goes to one as well in order to stay at the cooperation state in the future. Leniency bias improves incentives and associates with not only current principal and agent's incentives, but also future performance, which is consistent with theoretical implication in Zabojnik (2014) and empirical evidence in Bol (2011). Our results also resonate well with the evidence presented by Sebald and Walzl (2014). They demonstrate in a laboratory experiment that agents tend to impose punishment when the feedback from principals is below their subjective self-evaluations and in turn principals report more upwardly biased evaluations.

The existing theoretic literature has mainly focused upon leniency bias in a principal-supervisor-agent environment. Following the seminal work of Prendergast and Topel (1996), leniency bias is studied in the framework of optimal contract in Lee and Persson (2010), Grund and Przemeck (2012) and Giebe and Gürtler (2012). Most studies share the notion that the supervisor is not the residual claimant and that leniency bias arises from supervisor's altruistic preferences. Giebe and Gürtler (2012) consider a static contract where the supervisor's type, neutral or altruistic, is private information. It is costly for the principal to provide incentives for the altruistic supervisor to be truthfully because he has to be rewarded for reporting that the agent's effort is low. At the same time, the principal has to leave some rents to the neutral supervisor as well in order to prevent him from pretending to be the altruistic type. Hence, if the probability with which the supervisor is altruistic is sufficiently low, it is optimal not to provide incentives for the altruistic supervisor to be truthfully. Contrary to the traditional prediction that leniency bias is detrimental for optimal contracting, as in Rynes et al. (2005) and Marchegiani et al. (2016), the implication of Giebe and Gürtler (2012) is similar to ours in that it can be optimal not to eliminate leniency bias when designing the optimal contract. However, our work differs from this literature by studying a dynamic environment and showing that even as the residual claimant, the principal has incentives to report the agent's performance leniently.

This paper is closely related to the literature on repeated moral hazard with private evaluation. In the standard repeated moral hazard problem, the agent's output is publicly observable to both parties. As a consequence, no efficiency is lost if we restrict the agent to public strategies, which only depend on the public history of outputs. This is true no matter whether the agent's output is verifiable by outsiders, as in Spear and Srivastava (1987), or not, as in Baker et al. (1994).

When the agent's output is not publicly observable, Levin (2003) introduces a repeated principal-agent relationship in which the principal privately observes and truthfully reports the output. He shows that the agent is motived by the threat of termination. Macleod (2003) extends this analysis to a more general framework that each party receives a private signal about the agent's output. Macleod (2003) then studies the optimal static contract with variable degrees of signal correlation and shows that as long as the agent's signal is informative about the principal's, the agent can use it to provide incentives for the principal to be truthful. Both papers restrict analysis to equilibria in which private information is revealed every period. After that, Fuchs (2007) analyzes a class of equilibria in which the agent uses pure strategy and the principal aggregates the information acquired in a specific length of periods before evaluating the agent. Chan and Zheng (2011) and Chen et al. (2016) extend Fuchs (2007) to a general case where the agent's self-evaluation is correlated with the principal's evaluation of his performance. In such an environment, Maestri (2012) shows that bonus-payments equilibria always dominate efficiency-wage equilibria when both parties are patient. In addition to the quality of information, MacLeod and Tan (2017) explore authority contracts and sales contracts to emphasize the importance of the order of information revelation.

We extend Macleod (2003) to the dynamic environment in which the principal privately observes and publicly reports the agent's output, as in Levin (2003) and Fuchs (2007). This is an infinitely repeated game with private monitoring. We explicitly solve for the optimal PPE in such an environment. The optimal contract is static instead of dynamic, which generalizes the static contract result of Levin (2003) without imposing the constraint that the principal has to report truthfully. By allowing both parties with mixed strategy, we show that the agent's optimal public strategy is pure instead of mixed, which generalizes Fuchs (2007) without the constraint that the agent has to use a pure strategy. As for the efficiency, payoffs in review contracts approach the Pareto frontier as the discount factor goes to one, either by a class of efficiency-wage equilibria in Fuchs (2007) where the principal uses his private information to review the agent's performance, or a class of bonus-payments equilibria in Maestri (2012) where the agent evaluates the principal's performance. In our paper, however, we explicitly solve the optimal PPE for any given discount factor.

The rest of the paper is organized as follows: Section 2 sets up the model. Section 3 constructs and analyzes the optimal perfect public equilibrium. Finally, section 4 concludes.

## 2 Setup

Time is indexed by $t = 0, 1, \cdots$. There are an agent and a principal who are both risk neutral. From time to time, we refer to the agent as he and the principal as she. The principal has access to a project demanding the agent's effort as input. Given the agent's effort in period $t$ denoted by $e_t \in E \equiv \{0, 1\}$, his output in period $t$ denoted by $\theta_t \in \Theta \equiv \{\theta_L, \theta_H\}$ with $\theta_L < \theta_H$ is drawn from a time-invariant distribution

$$\pi(\theta_t = \theta_H \mid e_t) \equiv \begin{cases} q\,, \text{ if } \ e_t = 0 \\ p\,, \text{ if } \ e_t = 1 \end{cases} \text{ with } 0 < q < p < 1. \tag{1}$$

Shirking $e_t = 0$ is costless, but exerting effort $e_t = 1$ incurs a fixed cost $c > 0$ such that

$$\phi(e_t) \equiv \begin{cases} 0\,, \text{ if } \ e_t = 0 \\ c\,, \text{ if } \ e_t = 1 \end{cases}$$

is the agent's cost function of effort. Assume

$$\underline{\theta} \equiv (1-q)\theta_L + q\theta_H < (1-p)\theta_L + p\theta_H - c \equiv \overline{\theta} - c \tag{2}$$

which implies that the agent's effort is productive. The agent maximizes

$$(1-\delta)\mathbb{E}_\tau \left[ \sum_{t=\tau}^{\infty} \delta^{t-\tau}(w_t - \phi(e_t)) \right],$$

and the principal maximizes

$$(1-\delta)\mathbb{E}_\tau \left[ \sum_{t=\tau}^{\infty} \delta^{t-\tau}(\theta_t - w_t) \right]$$

where $\delta \in [0, 1)$ is the common discount factor, and $w_t \in \mathbb{R}$ is the principal's transfer to the agent in period $t$[1].

The agent's effort is unobservable to the principal. Due to (1), the principal is unable to infer with certainty whether the agent shirks or exerts effort from his output. In addition, the agent's output is privately observed by the principal.

In each period, the agent either shirks or exerts effort, then the principal reports (truthfully or not) the output. Hence, the agent's action set is $E$, and the principal's action set is $\Theta^2$. The principal's action $(\vartheta, \vartheta') \in \Theta^2$ implies that she reports $\vartheta$ ($\vartheta'$) when the true output is $\theta_L$ ($\theta_H$).

At the beginning of period $t$, the history of reported outputs $\vartheta^t \equiv (\vartheta_0, \cdots, \vartheta_{t-1}) \in \Theta^t$ is public information. Besides that, the agent has his private history of efforts $e^t \equiv (e_0, \cdots, e_{t-1}) \in E^t$, and the principal has her private history of true outputs $\theta^t \equiv (\theta_0, \cdots, \theta_{t-1}) \in \Theta^t$. Therefore, the agent's complete history is $(\vartheta^t, e^t)$, and the principal's complete history is $(\vartheta^t, \theta^t)$.

A contract is a mapping

$$w : \bigcup_{t=0}^{\infty} \Theta^t \to \mathbb{R}^2$$

as a transfer scheme contingent on public histories, which is enforceable by a third party[2]. Denote

[1]If $w_t < 0$, then the transfer goes from the agent to the principal. However, in the equilirbium constructed, the agent never pays the principal under any circumstances.

[2]In the equilibrium constructed, neither the agent nor the principal has incentives to leave the ongoing relationship under any circumstances. However, it is easier to start with this assumption.

the set of possible contracts by $W$. The agent's behavioral effort strategy is a mapping

$$s^A : \bigcup_{t=0}^{\infty} \left(\Theta^t \times E^t\right) \to \Delta(E),$$

and the principal's behavioral report strategy is a mapping

$$s^P : \bigcup_{t=0}^{\infty} \left(\Theta^t \times \Theta^t\right) \to (\Delta\Theta)^2[3].$$

Denote the set of the agent's possible strategies by $S^A$, and the set of the principal's possible strategies by $S^P$. Hence, a strategy profile is denoted by $s \equiv (s^A, s^P) \in S \equiv S^A \times S^P$.

Denote $\omega \mid \vartheta^t \in W$ as the continuation contract following public history $\vartheta^t$, $s^A \mid (\vartheta^t, e^t) \in S^A$ as the agent's continuation strategy following his history $(\vartheta^t, e^t)$, and $s^P \mid (\vartheta^t, \theta^t) \in S^P$ as the principal's continuation strategy following her history $(\vartheta^t, \theta^t)$. Denote $w[\vartheta^t](\vartheta_t)$ as the transfer in period $t$ contingent on reported output $\vartheta_t$ following public history $\vartheta^t$, $s^A[\vartheta^t, e^t](e_t)$ as the probability of the agent with history $(\vartheta^t, e^t)$ exerting effort $e_t$ in period $t$, and $s^P[\vartheta^t, \theta^t; \theta_t](\vartheta_t)$ as the probability of the principal with history $(\vartheta^t, \theta^t)$ reporting $\vartheta_t$ in period $t$ when observing true output $\theta_t$.

Denote $U^A(s, \omega)$ and $U^P(s, \omega)$ as the normalized expected discounted payoffs for the agent and the principal respectively given (continuation) strategy profile $s$ and (continuation) contract $\omega$.

Given strategy profile $s$, denote $\varphi[\vartheta^t, e^t; s](\theta^t) \in [0, 1]$ as the probability assigned by the agent with history $(\vartheta^t, e^t)$ to the principal having private history $\theta^t$ (or having complete history $(\vartheta^t, \theta^t)$), and $\mu[\vartheta^t, \theta^t; s](e^t) \in [0, 1]$ as the probability assigned by the principal with history $(\vartheta^t, \theta^t)$ to the agent having private history $e^t$ (or having complete history $(\vartheta^t, e^t)$). Apparently,

$$\sum\nolimits_{\theta^t} \varphi[\vartheta^t, e^t; s](\theta^t) = 1 \text{ and } \sum\nolimits_{e^t} \mu[\vartheta^t, \theta^t; s](e^t) = 1.$$

Furthermore, at the beginning of period 0,

$$\varphi[\vartheta^0, e^0; s](\theta^0) = 1 \text{ and } \mu[\vartheta^0, \theta^0; s](e^0) = 1$$

where $\vartheta^0$, $e^0$ and $\theta^0$ are empty sets. The agent's and principal's beliefs evolve as follows,

$$\varphi[\vartheta^{t+1}, e^{t+1}; s](\theta^{t+1}) = \frac{\varphi[\vartheta^t, e^t; s](\theta^t) s^P[\vartheta^t, \theta^t; \theta_t](\vartheta_t) \pi(\theta_t \mid e_t)}{\sum_{\overline{\theta}^{t+1}} \varphi[\vartheta^t, e^t; s](\overline{\theta}^t) s^P[\vartheta^t, \overline{\theta}^t; \overline{\theta}_t](\vartheta_t) \pi(\overline{\theta}_t \mid e_t)} \tag{3}$$

$$\mu[\vartheta^{t+1}, \theta^{t+1}; s](e^{t+1}) = \frac{\mu[\vartheta^t, \theta^t; s](e^t) s^A[\vartheta^t, e^t](e_t) \pi(\theta_t \mid e_t)}{\sum_{\overline{e}^{t+1}} \mu[\vartheta^t, \theta^t; s](\overline{e}^t) s^A[\vartheta^t, \overline{e}^t](\overline{e}_t) \pi(\theta_t \mid \overline{e}_t)} \tag{4}$$

respectively. However, the denominator in (3) may be zero (not necessarily) off the equilibrium path because the full support assumption is not satisfied. For instance, if the principal's strategy is to report the low output regardless of the true output, then the agent's belief is not well-defined by (3) after the principal reports the high output. If (3) is not well-defined for some $(\vartheta^{t+1}, e^{t+1})$, then we

[3]Or equivalently, $\Delta(\Theta^2)$.

define

$$\varphi[\vartheta^{t+1}, e^{t+1}; s](\theta^{t+1}) = \prod_{\tau=0}^{t} \pi(\theta_\tau \mid e_\tau) \tag{5}$$

instead. In addition, following $(\vartheta^{t+1}, e^{t+1})$, the agent's belief is defined by (5) as well. In words, as long as the agent detects that the principal has deviated from her strategy $s^P$, his belief becomes independent of the principal's reports thereafter. Given any strategy profile $s$, $\mu[\vartheta^{t+1}, \theta^{t+1}; s](e^{t+1})$ is well-defined even off the equilibrium path due to (1), which implies that the agent's potential deviations are undetectable to the principal. It can be shown that $((s^A, s^P), (\varphi, \mu))$ is consistent, as defined in Fudenberg and Tirole (1991).

**Definition 1** *A strategy profile $(s^A, s^P)$ is a sequential equilibrium (SE) with respect to $w$ if*

$$s^A \mid (\vartheta^t, e^t) \in \arg\max_{\tilde{s}^A \in S^A} \left\{ \sum\nolimits_{\theta^t} \varphi[\vartheta^t, e^t; s](\theta^t) U^A((\tilde{s}^A, s^P \mid (\vartheta^t, \theta^t)), w \mid \vartheta^t) \right\} \ \forall \vartheta^t, e^t, \tag{6}$$

$$s^P \mid (\vartheta^t, \theta^t) \in \arg\max_{\tilde{s}^P \in S^P} \left\{ \sum\nolimits_{e^t} \mu[\vartheta^t, \theta^t; s](e^t) U^P((s^A \mid (\vartheta^t, e^t), \tilde{s}^P), w \mid \vartheta^t) \right\} \ \forall \vartheta^t, \theta^t, \tag{7}$$

*while $(\varphi, \mu)$ is defined by (3)-(5).*

# 3 Analysis

Following Fudenberg, Levine and Maskin (1994), a perfect public equilibrium (PPE) is defined as a SE in which the agent's strategy $s^A$ is independent of his private history $e^t$, and the principal's strategy $s^P$ is independent of her private history $\theta^t$ [4] .

## 3.1 Static Contracting

**Definition 2** *A contract $w$ is static if $w[\vartheta^t](\vartheta_t)$ is independent of $\vartheta^t$.*

Given static contract $w$ which can be fully characterized by two wages $w_L$ and $w_H$, denote $V(w)$ as the set of feasible payoff pairs. Without loss of generality, assume $w_L \le w_H$. The agent's minmax payoff is defined by

$$\min_{r \in [0,1]^2} \max_{e \in [0,1]} \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} w_H \\ w_L \end{bmatrix} - ec,$$

and the principal's minmax payoff is defined by

$$\min_{e \in [0,1]} \max_{r \in [0,1]^2} \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \left( \begin{bmatrix} \theta_H \\ \theta_L \end{bmatrix} + \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} -w_H \\ -w_L \end{bmatrix} \right)$$

where $e \in [0,1]$ is the probability of the agent exerting effort, and $r \equiv (r_L, r_H) \in [0,1]^2$ are the probabilities of the principal reporting the high output when the true output is low and high respectively.

[4] We are able to show that if the agent uses a pure strategy, then whether or not the agent bases his current action on his private history is not going to change the set of payoff pairs attainable. A more complex case in which the agent uses a mixed private strategy is studied in another paper of ours.

If $r = (0, 1)$, then the principal reports truthfully. Therefore, the minmax payoffs for the agent and the principal are $w_L$ and $\underline{\theta} - w_L$ respectively. Denote

$$V^*(w) \equiv \{(U^A, U^P) \in V(w) \mid U^A \geq w_L \text{ and } U^P \geq \underline{\theta} - w_L\}$$

as the set of individually rational payoff pairs. Moreover, $U^A + U^P$ is used to measure efficiency because both parties are risk neutral.

Define a mapping $\Pi : 2^{\mathbb{R}^2} \to 2^{\mathbb{R}^2}$ as

$$\Pi(X) = \left\{ (U^A, U^P) \;\middle|\; \begin{array}{c} \exists U_L, U_H \in co(X)[5],\ e \in [0,1] \text{ and } r \in [0,1]^2 \\ \text{such that} \\ U^A = U^A((e,r), w; U_L, U_H) \text{ and } U^P = U^P((e,r), w; U_L, U_H) \\ e \in \underset{e' \in [0,1]}{\arg\max} U^A((e', r), w; U_L, U_H) \text{ and } r \in \underset{r' \in [0,1]^2}{\arg\max} U^P((e, r'), w; U_L, U_H) \end{array} \right\} \tag{8}$$

where

$$U^A((e,r), w; U_L, U_H) = \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} (1-\delta)w_H + \delta U_H^A \\ (1-\delta)w_L + \delta U_L^A \end{bmatrix} - e(1-\delta)c,$$

$$U^P((e,r), w; U_L, U_H) = \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \left( \begin{bmatrix} (1-\delta)\theta_H \\ (1-\delta)\theta_L \end{bmatrix} + \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} -(1-\delta)w_H + \delta U_H^P \\ -(1-\delta)w_L + \delta U_L^P \end{bmatrix} \right).$$

Hence, $\Pi^\infty(V(w))$ is the set of PPE (with respect to $w$) payoff pairs according to Abreu, Pearce and Stacchetti (1990).

**Proposition 1** *Given static contract $w$, denote*

$$\overline{U}(w) \equiv \max U^A + U^P \ \ s.t.\ \ (U^A, U^P) \in \Pi^\infty(V(w))$$

*as the maximum total payoff attainable by PPE with respect to static contract $w$. Then,*

$$\overline{U}(w) \begin{cases} = \underline{\theta} & , \text{ for } \delta \in [0, \underline{\delta}(w_H - w_L)) \\ \\ \leq \overline{\theta} - (1-q)c/(p-q) & , \text{ for } \delta \in [\underline{\delta}(w_H - w_L), 1) \end{cases}$$

*where*

$$\underline{\delta}(w_H - w_L) \equiv \min \left\{ \frac{\max\{c/(p-q), w_H - w_L\}}{\max\{c/(p-q), w_H - w_L\} + (p-q)(\theta_H - \theta_L) - (1-q)c/(p-q)}, 1 \right\}. \tag{9}$$

**Proof.** See Appendix A. ■

Given static contract $w$, the blue parallelogram in Figure 1(a) represents $V(w)$ - the set of feasible payoff pairs. And the shadowed area represents $\Pi^\infty(V(w))$ - the set of PPE (with respect to $w$) payoff pairs, which includes $(w_L, \underline{\theta} - w_L)$ because the agent shirks and the principal reports the low output

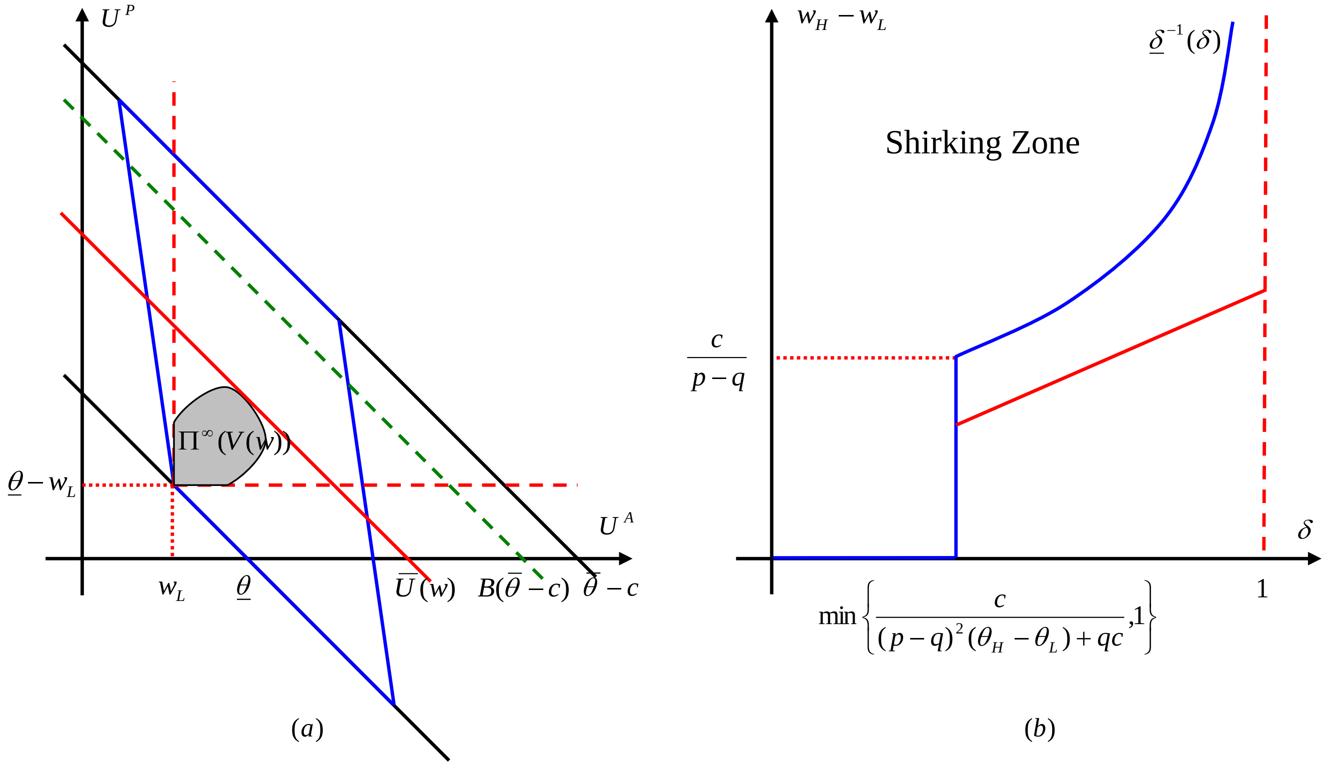


**Figure 1**

regardless of the true output is a Nash equilibrium in the stage game. We generalize the method of Radner, Myerson and Maskin (1986) (RMM thereafter) by considering the fact that any continuation payoff pair must Pareto dominate the minmax payoff pair $(w_L, \underline{\theta} - w_L)$. As a result, the upper bound derived is not completely independent of the discount factor as in RMM.

Proposition 1 suggests that what matters is the bonus $w_H - w_L$, not the base wage $w_L$. Note that $\underline{\delta}(w_H - w_L)$ goes to 1 as $w_H - w_L$ goes to infinity. Thus, given any discount factor, if the bonus is large enough, then $\delta < \underline{\delta}(w_H - w_L)$, which implies $\overline{U}(w) = \underline{\theta}$, which in turn implies that the agent never exerts effort. Specifically, as illustrated in Figure 1(b), given any discount factor, if the bonus is greater than $\underline{\delta}^{-1}(\delta)$ (above the blue curve), then the agent never exerts effort. This is a little bit counter-intuitive. Recall that in the case of the agent's output being publicly observable, larger the bonus, more incentives for the agent to exert effort. Specifically, the agent exerts effort if and only if the bonus is greater than $\frac{c}{p-q}$[6]. The logic is as follows: when the agent's output is privately observed by the principal as in this paper, the agent's incentives to exert effort depend upon not only how large the bonus is, but also whether the principal is willing to deliver the bonus by reporting truthfully when the agent's output is high. However, larger the bonus, more incentives for the principal to report the agent's high output as low in order to save money. Therefore, when

[6]The agent is indifferent between shirking and exerting effort if the bonus is equal to $\frac{c}{p-q}$.

designing the optimal contract, the bonus has to be large enough in order for the agent to exert effort, but at the same time, small enough in order for the principal to be truthful.

**Corollary 1** *Suppose*

$$\delta < \min\left\{\frac{c}{(p-q)^2(\theta_H-\theta_L)+qc}, 1\right\}. \tag{10}$$

*Then, given any static contract, there exists a unique PPE in which the agent always shirks and the principal always reports the low output regardless of the true output. As a result, the total payoff is the lowest at $\underline{\theta}$.*

Corollary 1 suggests that if both parties are not patient enough, then the unique PPE is simply a repetition of the Nash equilibrium in the stage game.

Suppose

$$\left(\frac{p-q}{1-q}\right)\left[\frac{(p-q)(\theta_H-\theta_L)}{c}\right] \le 1. \tag{11}$$

Then the RHS of (10) is greater than one, which implies that for all $\delta \in [0,1)$, the result in Corollary 1 holds. The first term $\frac{p-q}{1-q} \in [0,1]$ is a percentage by which the agent is able to lower the probability of the output being low by exerting effort. Higher the percentage, easier for the principal to detect whether the agent shirks. For instance, if $p=1$ (therefore $\frac{p-q}{1-q}=1$), then the principal knows for sure that the agent has been shirking whenever the output turns out to be low. And the second term $\frac{(p-q)(\theta_H-\theta_L)}{c} \ge 1$ is a surplus increase per unit of cost of the agent exerting effort. Under the condition of (11), any total payoff when the agent exerts effort with strictly positive probability is no greater than the total payoff when he shirks. As a result, by the iteration, the upper bound on the maximum total payoff attainable by PPE converges to the lowest at $\underline{\theta}$.

**Proposition 2** *(The optimal PPE with Leniency Bias) Suppose*

$$\frac{c}{(p-q)^2(\theta_H-\theta_L)+qc} \le \delta < 1.$$

*Define static contract $w$ with $w_L = 0$ and $w_H = \delta(p-q)(\theta_H-\theta_L)$. Define an automaton with two states: a cooperation state $C$ in which the agent exerts effort and the principal reports the high output with probability $1-\frac{(1-\delta)c}{\delta[(p-q)^2(\theta_H-\theta_L)-(1-q)c]}$ (1) when the true output is low (high), and a defection state $D$ in which the agent shirks and the principal reports the low output regardless of the true output. Let $C$ be the initial state. The transition from $C$ to $D$ occurs if and only if the low output is reported. But $C$ is inaccessible to $D$ which implies that as far as $D$ is reached, it prevails forever. Then, this is a PPE with respect to $w$. As a result, the total payoff is $\overline{\theta}-\frac{1-q}{p-q}c$ which implies that it is optimal according to Proposition 1.*

**Proof.** See Appendix B. ■

The defection state features the Nash equilibrium in the stage game. At the cooperation state, the principal is indifferent between reporting the low output and reporting the high output regardless of the true output. Hence, the probability with which the principal reports the agent's low output

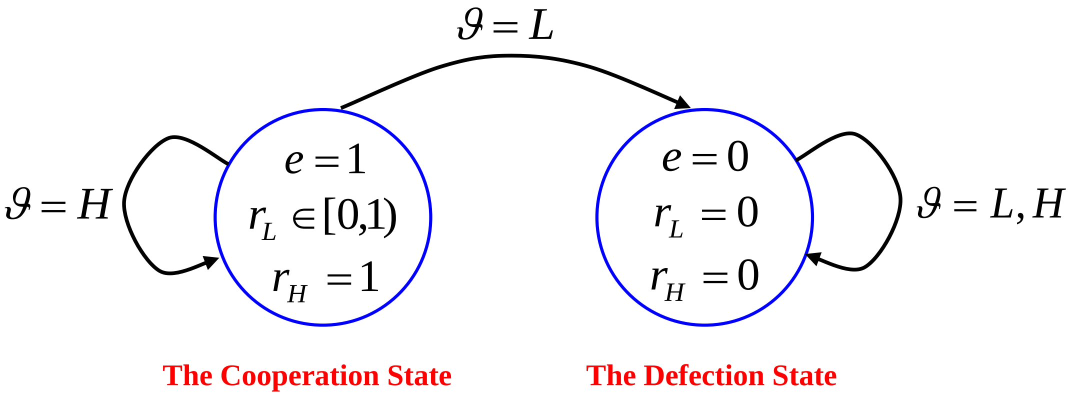


**Figure 2**

truthfully - $1 - r_L$ is chosen to be large enough such that the agent has incentives to exert effort, but not too large such that unnecessary mutual punishments at the defection state are triggered. As a result, the agent is also indifferent between shirking and exerting effort at the cooperation state.

**Corollary 2** *Define leniency bias as the probability with which the principal reports the agent's low output as high:*

$$1 - \frac{(1-\delta)c}{\delta[(p-q)^2(\theta_H - \theta_L) - (1-q)c]}.$$

*Then (i) as the discount factor goes to one, the probability with which the principal overreports the agent's output goes to one as well;*

*(ii) more costly for the agent to exert effort, less likely for the principal to overreport the agent's output.*

At the cooperation state, the agent is always rewarded for the high output, but *not* always punished for the low output (with some positive probability, the principal reports the agent's low output as high). The logic is as follows: on one hand, whenever the agent's output is low, the principal has to report the agent's output truthfully (at least with some positive probability) in order to provide incentives for the agent to exert effort ex ante. On the other hand, whenever the principal reports the low output, the agent has to punish the principal by exerting less effort in the future in order to provide incentives for the principal to be truthful ex ante. Therefore, as far as the efficiency is concerned, the probability with which the principal reports the agent's low output truthfully has to be large enough such that the agent has incentives to exert effort, but not too large such that unnecessary mutual punishments at the defection state are triggered. As suggested by

Prendergast (1999), this phenomenon is well documented in empirical studies as leniency bias, which implies that an supervisor tends to overstate an subordinate's performance.

An implication of Proposition 2 is that leniency bias is not necessarily eliminated by incentive contracts. In this model, leniency bias is the outcome of an optimal contract under subjective evaluations. The principal has incentive to inflate the agent's performance even as the residual claimant. The agent has incentive to exert effort until the low output is reported. In the optimal PPE illustrated above, Corollary 2 suggests that the magnitude of leniency bias increases with the discount factor and decreases with the cost for the agent to exert effort. Specifically, if the agent becomes more patient, then the principal would express more leniency to avoid retaliation without demotivating the (more patient) agent. If it becomes more costly for the agent to exert effort, then the principal has to express less leniency in order to motivate the agent to exert (more costly) effort. Furthermore, if the discount factor goes to one, the probability with which the principal reports the agent's low output truthfully has to be lower enough such that the transition from C to D is less likely triggered, which implies leniency bias goes to one as well.

**Corollary 3** *Suppose that both the agent's reservation payoff and the principal's reservation payoff outside the relationship are zero. Then neither the agent nor the principal has incentives to leave the ongoing relationship under any circumstances.*

On the agent's part, the relationship is self-enforcing as long as the agent's payoff by staying in the relationship is greater or equal to his reservation payoff, which is zero. On the principal's part, if the principal refuses to pay the bonus after reporting the high output, then the agent walks away from the relationship. Hence, the relationship is self-enforcing as long as the principal's payoff by paying the bonus is greater or equal to her payoff by *not* paying the bonus[7]. It is straightforward to show that both conditions are satisfied such that neither the agent nor the principal has incentives to leave the ongoing relationship under any circumstances.

The red interval with slope $(p-q)(\theta_H - \theta_L)$ in Figure 1(b) represents the optimal static contract characterized by $w_H - w_L$. It is interesting to observe that when the discount factor is just slightly larger than $\frac{c}{(p-q)^2(\theta_H-\theta_L)+qc}$, the agent exerts effort even when $w_H - w_L$ is strictly less than $\frac{c}{p-q}$. On the contrary, if the agent's output is publicly observable, the agent never exerts effort if $w_H - w_L < \frac{c}{p-q}$. The difference is that when the agent's output is publicly observable, the agent's expected payoff from shirking is $(1-q)w_L + qw_H$ guaranteed. But in this model, the principal can punish the agent even further by reporting low output $\theta_L$ regardless of true output $\theta_t$ so that the agent's expected payoff from shirking is $w_L$ instead. In some sense, the principal is able to terminate the agent[8]. Therefore, as long as the relationship is still valuable to the agent in the sense that the bonus is not too small, or specifically $(1-p)w_L + pw_H \geq c$[9], the agent has incentives to exert effort in order to stay in.

[7]In the equilibrium constructed, the principal's payoff by *not* paying the bonus is equal to her payoff by simply reporting the low output in the first place.

[8]This implies that allowing the principal to be able to terminate the agent explicitly in this model may not be necessary. Termination basically serves as a public signal for both parties to go to the defection state.

[9]This requires $w_H - w_L > \frac{c}{p}$ which is apparently weaker than $w_H - w_L > \frac{c}{p-q}$.

## 3.2 Dynamic Contracting

**Proposition 3** *Denote*

$$\overline{U} \equiv \max U^A + U^P \;\; s.t. \;\; (U^A, U^P) \in \underset{w \in W}{\cup} \Gamma^\infty(V(w))$$

*as the maximum total payoff attainable by PPE. Then,*

$$\overline{U} = \begin{cases} \underline{\theta} & , \textit{for } \; \delta \in [0, c/[(p-q)^2(\theta_H - \theta_L) + qc]) \\ \overline{\theta} - (1-q)c/(p-q) & , \textit{for } \; \delta \in [c/[(p-q)^2(\theta_H - \theta_L) + qc], 1) \end{cases} .$$

**Proof.** See Appendix C. ■

Proposition 3 generalizes the static contract result of Levin (2003) without imposing the constraint that the principal reports truthfully[10]. It turns out to be easier to derive the set of PPE payoff pairs with respect to some dynamic contract directly, instead of the set of PPE payoff pairs with respect to a certain dynamic contract. The way to do that is to imagine that there exists a third player called contract generator, for instance, besides the agent and the principal. The contract generator's action set is $\mathbb{R}^2$. And its strategy is a mapping

$$w : \overset{\infty}{\underset{t=0}{\cup}} \Theta^t \to \mathbb{R}^2.$$

As a result, the set of PPE payoff pairs is characterized as an area between two parallel lines because if $(U^A, U^P)$ is in it, then any $(\widetilde{U}^A, \widetilde{U}^P)$ with $\widetilde{U}^A + \widetilde{U}^P = U^A + U^P$ is in it too. All we have to do is to add a constant transfer without interfering incentives for the agent and the principal.

# 4 Conclusion

We have studied an infinitely repeated principal-agent problem in which the principal privately observes and publicly reports the agent's output. We solve for the optimal PPE explicitly. We show that leniency bias arises in equilibrium even if the principal is the residual claimant. For future research, it would be interesting to see how the agent is able to generate private information on the equilibrium path with mixed strategies in order to provide incentives for the principal to be truthful.

[10]But the role of dynamic contracting is unclear when it comes to SE with the agent's mixed strategies. See Fuchs (2007) for a static contract result with the agent's pure strategies.

# Appendix

## A Proof of Proposition 1

Given static contract $w$, define a function $B : [\underline{\theta}, +\infty) \to \mathbb{R}$ as

$$B(x) = \max U^A + U^P \text{ s.t. } (U^A, U^P) \in \Gamma(\Lambda(x)) \tag{12}$$

where $\Lambda(x) \equiv \{(U^A, U^P) \mid U^A \geq w_L, U^P \geq \underline{\theta} - w_L \text{ and } U^A + U^P \leq x\}$.

The proof takes three steps.

**Step 1** We show $\overline{U}(w) \leq B^\infty(\overline{\theta} - c)$.

Since $\overline{\theta} - c$ is the maximum total payoff, it follows that $\Gamma^\infty(V(w)) \subseteq V^*(w) \subseteq \Lambda(\overline{\theta} - c)$. Hence, $\Gamma^\infty(V(w)) \subseteq \Gamma(\Lambda(\overline{\theta} - c))$ because $\Gamma$ is monotonic. By the definition of $\overline{U}(w)$ and $B(\overline{\theta} - c)$, we conclude that $\overline{U}(w) \leq B(\overline{\theta} - c) \leq \overline{\theta} - c$. By iteration, we can construct a decreasing sequence $\left\{B^n(\overline{\theta} - c) \geq \overline{U}(w)\right\}_{n=0}^\infty$, which implies $\overline{U}(w) \leq B^n(\overline{\theta} - c) \leq \cdots \leq B^2(\overline{\theta} - c) \leq B(\overline{\theta} - c) \leq \overline{\theta} - c$. Hence, $B^\infty(\overline{\theta} - c)$ is an upper bound of $\overline{U}(w)$.

**Step 2** We solve for $B(x)$.

**Lemma 1** *(a) If $(p-q)^2(\theta_H - \theta_L) < (1-q)c$, then $B(x) = \delta x + (1-\delta)\underline{\theta}$;*

*(b) If $(p-q)^2(\theta_H - \theta_L) \geq (1-q)c$, then*

$$B(x) = \begin{cases} \delta x + (1-\delta)\underline{\theta} & , \textit{for } x < \underline{\theta} + (1-\delta)\max\left\{c/(p-q), w_H - w_L\right\}/\delta \\ \\ \delta x + (1-\delta)[\overline{\theta} - (1-q)c/(p-q)] \, , & \textit{otherwise} \end{cases} .$$

**Proof.** Rewrite the optimization problem as follows,

$$B(x) = \max_{e,r,U_L,U_H} \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \left( \begin{bmatrix} (1-\delta)\theta_H \\ (1-\delta)\theta_L \end{bmatrix} + \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} \delta(U_H^A + U_H^P) \\ \delta(U_L^A + U_L^P) \end{bmatrix} \right) - e(1-\delta)c$$

subject to

$$e \in \underset{e' \in [0,1]}{\arg\max} U^A((e', r), w; U_L, U_H) \tag{13}$$

$$r \in \underset{r' \in [0,1]^2}{\arg\max} U^P((e, r'), w; U_L, U_H) \tag{14}$$

$$U_L, U_H \in \Lambda(x). \tag{15}$$

We proceed by considering two auxiliary optimization problems: one with the additional constraint $e = 0$ and one with the additional constraint $e > 0$. Therefore, $B(x)$ can be calculated by comparing the results from these two complementary optimization problems.

**Case 1** $e = 0$.

It's straightforward to show $B_1(x) = \delta x + (1-\delta)\underline{\theta}$ for $U_L^A + U_L^P = U_H^A + U_H^P = x$.

**Case 2** $e > 0$.

Note that $e > 0$ only if

$$U_H^P - U_L^P = (1-\delta)(w_H - w_L)/\delta \tag{16}$$

$$(r_H - r_L)(p-q)[(1-\delta)(w_H - w_L) + \delta(U_H^A - U_L^A)] \geq (1-\delta)c \tag{17}$$

where (16) holds because otherwise the principal reports the low/high output regardless of the true output so that the agent does not have incentives to exert effort, and (17) is derived from (13).

Therefore, we proceed by assuming $x - \underline{\theta} \geq (1-\delta)(w_H - w_L)/\delta$ because, otherwise, (16) can not hold. Without loss of generality, we assume

$$U_L^P = \underline{\theta} - w_L \text{ and } U_H^P = \underline{\theta} - w_L + (1-\delta)(w_H - w_L)/\delta \tag{18}$$

$$\max\left\{U_L^A + U_L^P, U_H^A + U_H^P\right\} = x. \tag{19}$$

(i) If $U_H^A + U_H^P = x$, we have

$$U_H^A = x - [\underline{\theta} - w_L + (1-\delta)(w_H - w_L)/\delta] \text{ and } U_L^A \in [w_L, x - (\underline{\theta} - w_L)]$$

which implies $(1-\delta)(w_H - w_L) + \delta(U_H^A - U_L^A) \in [0, \delta(x - \underline{\theta})]$. Therefore, $r_H > r_L$ according to (17) which implies

$$U_H^A - U_L^A \geq \frac{1-\delta}{\delta(p-q)(r_H - r_L)}c - \frac{1-\delta}{\delta}(w_H - w_L) \geq \frac{1-\delta}{\delta(p-q)}c - \frac{1-\delta}{\delta}(w_H - w_L).$$

The first equality always holds since otherwise $U_L^A$ can be increased without changing $e$, $r_L$ and $r_H$ resulting in a greater value of the objective function. So we can rewrite the optimization problem as follows,

$$\max_{e, r_L, r_H \in [0,1]} \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \left( \begin{bmatrix} (1-\delta)\theta_H \\ (1-\delta)\theta_L \end{bmatrix} + \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\left(x - \frac{1-\delta}{\delta(p-q)(r_H - r_L)}c\right) \end{bmatrix} \right) - e(1-\delta)c$$

subject to

$$\frac{1-\delta}{\delta(p-q)(r_H - r_L)}c \in [0, x - \underline{\theta}].$$

The constraint set is non-empty if and only if $x \geq \underline{\theta} + \frac{1-\delta}{\delta(p-q)}c$. As a result, the maximum value is $(1-\delta)(\overline{\theta} - \frac{1-q}{p-q}c) + \delta x$ for $e = r_H = 1$ and $r_L = 0$.

(ii) If $U_L^A + U_L^P = x$, we have

$$U_H^A \in \left[w_L, x - \left(\underline{\theta} - w_L + \frac{1-\delta}{\delta}(w_H - w_L)\right)\right] \text{ and } U_L^A = x - (\underline{\theta} - w_L)$$

which implies $(1-\delta)(w_H - w_L) + \delta(U_H^A - U_L^A) \in [(1-\delta)(w_H - w_L) - \delta(x - \underline{\theta}), 0]$. Therefore, $\rho_H < \rho_L$ according to (17) which implies

$$U_H^A - U_L^A \leq \frac{1-\delta}{\delta(p-q)(r_H - r_L)}c - \frac{1-\delta}{\delta}(w_H - w_L) \leq -\frac{1-\delta}{\delta(p-q)}c - \frac{1-\delta}{\delta}(w_H - w_L).$$

The first equality always holds since otherwise $U_H^A$ can be increased without changing $e$, $r_L$ and $r_H$ resulting in a greater value of the objective function. So we can rewrite the optimization problem as follows,

$$\max_{e,r_L,r_H\in[0,1]} \begin{bmatrix} e \\ 1-e \end{bmatrix}^\top \begin{bmatrix} p & 1-p \\ q & 1-q \end{bmatrix} \left( \begin{bmatrix} (1-\delta)\theta_H \\ (1-\delta)\theta_L \end{bmatrix} + \begin{bmatrix} r_H & 1-r_H \\ r_L & 1-r_L \end{bmatrix} \begin{bmatrix} \delta\left(x+\frac{1-\delta}{\delta(p-q)(r_H-r_L)}c\right) \\ \delta x \end{bmatrix} \right) - e(1-\delta)c$$

subject to

$$\frac{1-\delta}{\delta(p-q)(r_H-r_L)}c \in \left[\frac{1-\delta}{\delta}(w_H-w_L)-(x-\underline{\theta}), 0\right].$$

The constraint set is non-empty if and only if $x \geq \underline{\theta} + \frac{1-\delta}{\delta(p-q)}c + \frac{1-\delta}{\delta}(w_H - w_L)$. As a result, the maximum value is $(1-\delta)\left(\overline{\theta} - \frac{1-q}{p-q}c\right) + \delta x$ for $e = r_L = 1$ and $r_H = 0$.

Hence, we conclude $B_2(x) = (1-\delta)\left(\overline{\theta} - \frac{1-q}{p-q}c\right) + \delta x$ if $x \geq \underline{\theta} + \frac{1-\delta}{\delta(p-q)}c$.

Therefore,

$$B(x) = \begin{cases} \max\{B_1(x), B_2(x)\}, & \text{if } x \geq \underline{\theta} + (1-\delta)c/[\delta(p-q)] \\ B_1(x) & \text{, if otherwise} \end{cases}.$$

The result follows by algebra. ■

**Step 3** The function $B(x)$ is illustrated in Figure 3. Therefore, as long as $B(x) \geq x$ for $x = \underline{\theta} + (1-\delta)\max\{c/(p-q), w_H - w_L\}/\delta$, $B^\infty(\overline{\theta}-c) = \overline{\theta} - (1-q)c/(p-q)$. Otherwise, $B^\infty(\overline{\theta}-c) = \underline{\theta}$.

This completes the proof.

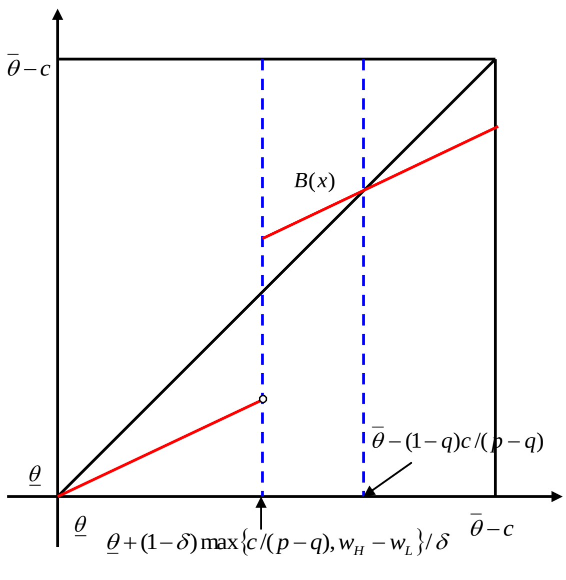


**Figure 3**

## B Proof of Proposition 2

Let $U^i(j)$ for $i \in \{A, P\}$ and $j \in \{C, D\}$ denote the player $i$'s NED payoff at state $j$. Apparently, we have $U^A(D) = w_L^*$ and $U^P(D) = \underline{\theta} - w_L^*$. Furthermore,

$$U^A(C) = (1-\delta)(\overline{w} - c) + \delta[pU^A(C) + (1-p)((1-r_L)U^A(D) + r_L U^A(C))]$$

$$U^P(C) = (1-\delta)(\overline{\theta} - \overline{w}) + \delta[pU^P(C) + (1-p)((1-r_L)U^P(D) + r_L U^P(C))]$$

where $\overline{w} = pw_H^* + (1-p)[(1-r_L)w_L^* + r_L w_H^*]$. By algebra, we have

$$U^A(C) = \frac{(1-\delta)(\overline{w} - c) + \delta(1-p)(1-r_L)w_L}{1 - \delta p - \delta(1-p)r_L}$$

$$U^P(C) = \frac{(1-\delta)(\overline{\theta} - \overline{w}) + \delta(1-p)(1-r_L)(\underline{\theta} - w_L)}{1 - \delta p - \delta(1-p)r_L}$$

so that

$$U^A(C) + U^P(C) = \frac{(1-\delta)(\overline{\theta} - c) + \delta(1-p)(1-r_L)\underline{\theta}}{1 - \delta p - \delta(1-p)r_L} = \overline{\theta} - \frac{1-q}{p-q}c$$

as claimed.

The incentive compatibility at state $D$ is straightforward. At state $C$, given the principal's strategy of reporting truthfully, the agent's NED payoff of shirking and exerting effort are

$$(1-\delta)\underline{w} + \delta[qU^A(C) + (1-q)((1-r_L)U^A(D) + r_L U^A(C))]$$

$$(1-\delta)(\overline{w} - c) + \delta[pU^A(C) + (1-p)((1-r_L)U^A(D) + r_L U^A(C))]$$

respectively which are equal by algebra where $\underline{w} = qw_H^* + (1-q)[(1-r_L)w_L^* + r_L w_H^*]$. So it's optimal for the agent to exert effort at State $C$. Also at state $C$, given the agent's strategy of exerting effort, the expected utilities for the principal to report the low output and the high output (regardless of the true output) are

$$-(1-\delta)w_L^* + \delta U^P(D) \text{ and } -(1-\delta)w_H^* + \delta U^P(C)$$

respectively which are equal by algebra. So it's optimal for the principal to report truthfully at state $C$. And $\delta \in \left[\frac{c}{(p-q)^2(\theta_H - \theta_L) + qc}, 1\right)$ guarantees $r_L \in [0, 1]$.

This completes the proof.

# C Proof of Proposition 3

For $x \in [\underline{\theta}, +\infty)$, $\Lambda_1(x) \equiv \{(U^A, U^P) \mid \underline{\theta} \leq U^A + U^P \leq x\}$. And define a mapping $\Pi_1 : 2^{\mathbb{R}^2} \to 2^{\mathbb{R}^2}$ as

$$\Pi_1(X) = \left\{ (U^A, U^P) \left| \begin{array}{c} \exists U_L, U_H \in co(X),\ w \in \mathbb{R}^2,\ e \in [0,1] \text{ and } r \in [0,1]^2 \\ \text{such that} \\ U^A = U^A((e,r), w; U_L, U_H) \text{ and } U^P = U^P((e,r), w; U_L, U_H) \\ e \in \underset{e' \in [0,1]}{\arg\max} U^A((e', r), w; U_L, U_H) \text{ and } r \in \underset{r' \in [0,1]^2}{\arg\max} U^P((e, r'), w; U_L, U_H) \end{array} \right. \right\}.$$

Now, the set of the feasible payoff pairs $V$ is defined as

$$V = \{(U^A, U^P) \mid \underline{\theta} \leq U^A + U^P \leq \overline{\theta} - c\}$$

because the payoffs are transferable between the agent and the principal. Therefore, there exists $x \in [\underline{\theta}, \overline{\theta} - c]$ such that

$$co(\Pi_1^\infty(V)) = \{(U^A, U^P) \mid \underline{\theta} \leq U^A + U^P \leq x\}$$

which implies that we can iterate on $x$ instead of the whole set. By applying the procedure in the proof of Proposition 2, we have

$$x = \begin{cases} \underline{\theta} & \text{, for } \delta \in \left(0, \frac{c}{(p-q)^2(\theta_H - \theta_L) + qc}\right) \\ \overline{\theta} - (1-q)c/(p-q) & \text{, for } \delta \in \left[\frac{c}{(p-q)^2(\theta_H - \theta_L) + qc}, 1\right) \end{cases}.$$

This completes the proof.

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